Probability and Random Processes

ECS 315

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11 Multiple Random Variables

Office Hours:

BKD, 6th floor of Sirindhralai building

Wednesday 14:00-15:30

Friday 14:00-15:30

Chapter 6 vs. Chapter 11

$P(A \cap B)$

Joint pmf

$$p_{X,Y}(x,y) = P[X = x, Y = y]$$

$A = [X = x]$

$B = [Y = y]$

Conditional pmf

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B|A)P(A)}{P(B)}$$

$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$$

$$= \frac{p_{Y|X}(y|x)p_X(x)}{p_Y(y)}$$

Events A and B are independent:

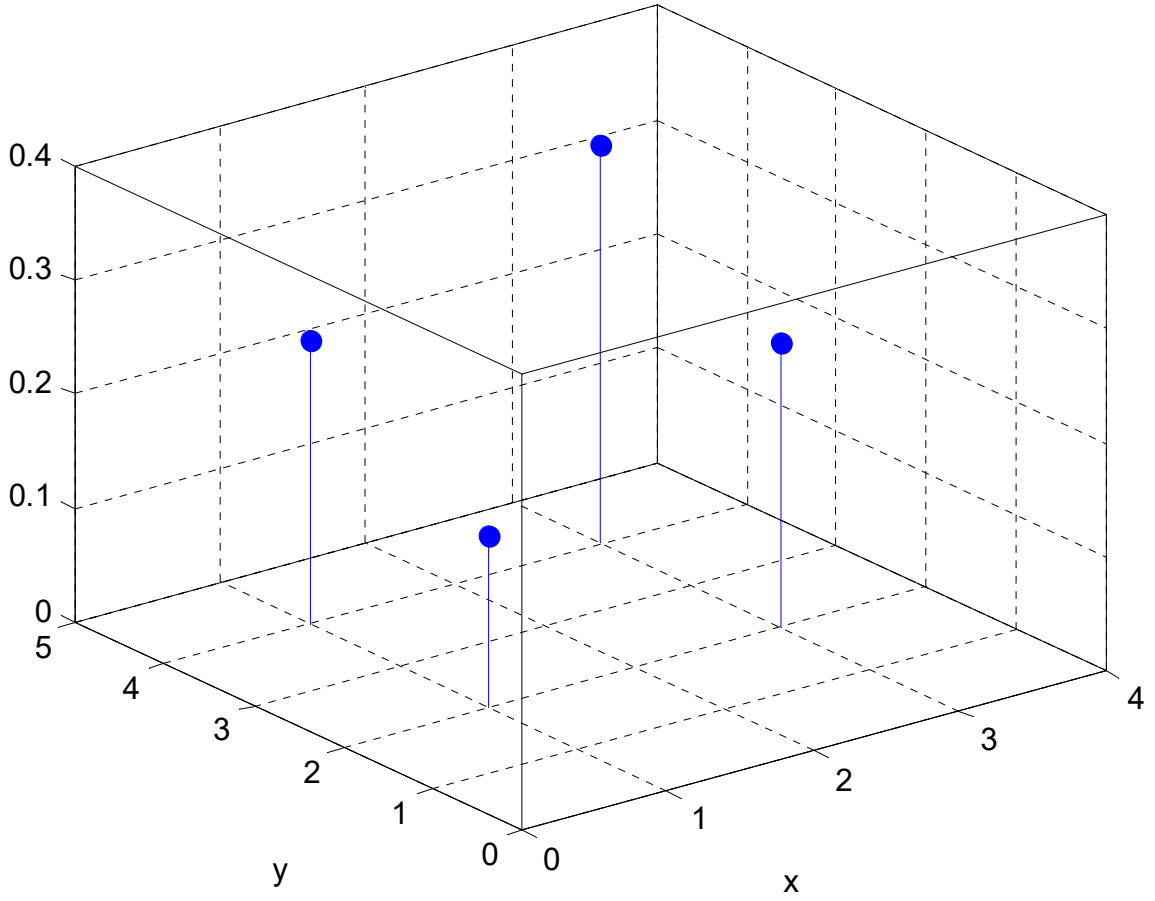
$$P(A \cap B) = P(A)P(B)$$

RVs X and Y are **independent**:

$$p_{X,Y}(x,y) = p_X(x)p_Y(y) \text{ for any } x \text{ and } y$$

Example: small joint pmf matrix Ex. 11.7

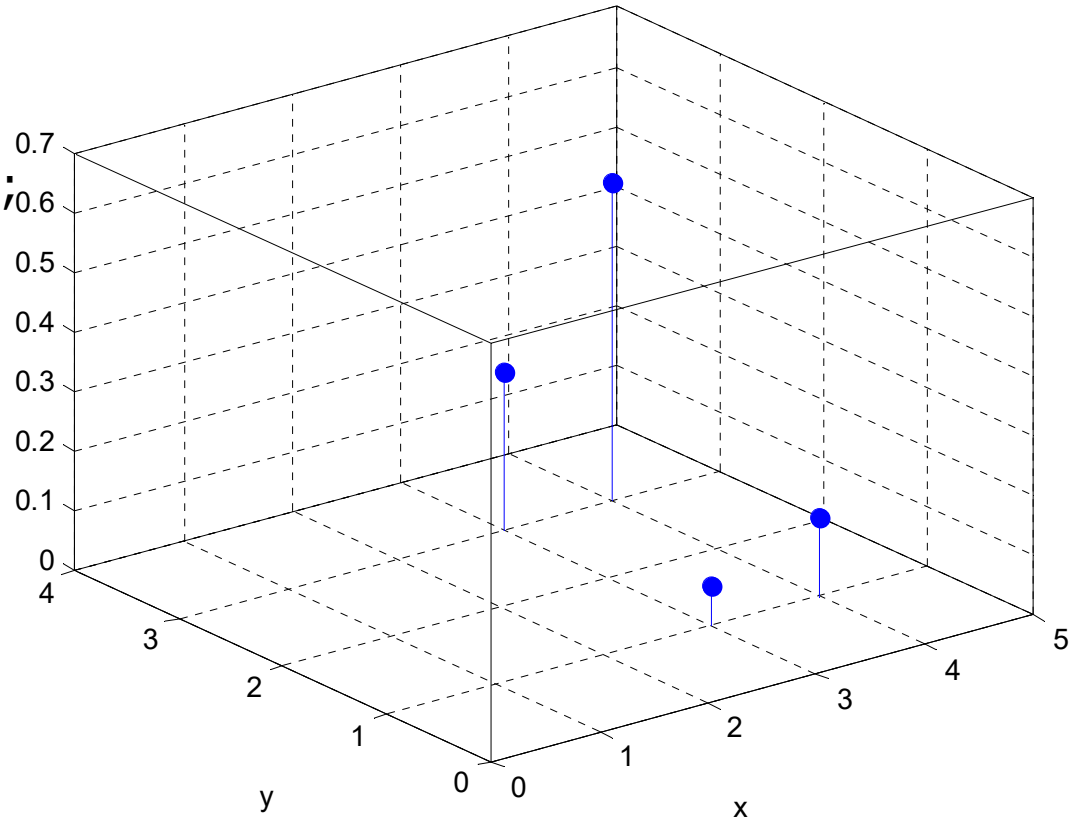
```
close all; clear all;
x = [1 3];
y = [2 4];
PXY = [3/20 5/20; 5/20 7/20];

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY,'filled')
xlim([0,4])
ylim([0,5])
xlabel('x')
ylabel('y')
```



(More)

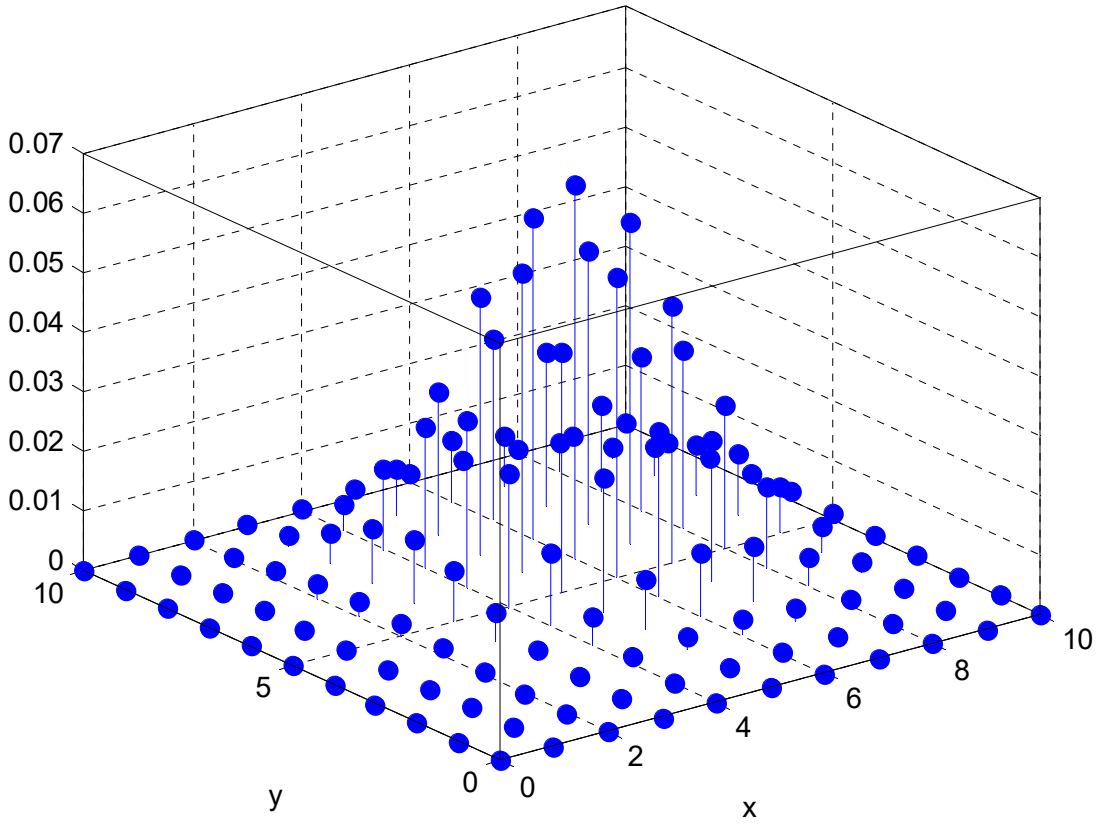
Example: small joint pmf matrix Ex. 11.26

```
close all; clear all;
x = [3 4];
y = [1 3];
PXY = [1/15 4/15; 2/15 8/15];

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY,'filled')
xlim([0,5])
ylim([0,4])
xlabel('x')
ylabel('y')
```



Example: large joint pmf matrix

```
close all; clear all;
n = 10; p = 3/5;
x = 0:n;
y = 0:n;

pX = binopdf(x,n,p);
pY = binopdf(y,n,p);

PXY = pX.'*pY;

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY, 'filled')
%mesh(X,Y,PXY)
%surf(X,Y,PXY)

xlabel('x')
ylabel('y')
```



Evaluation of Probability

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X + Y < 7]$

Evaluation of Probability

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- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X + Y < 7]$

Step 1: Find the pairs (x,y) that satisfy the condition "x+y < 7"

One way to do this is to first construct the matrix of $x+y$.

$x + y$

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

Evaluation of Probability

- Consider two random variables X and Y.
- Suppose their **joint pmf matrix** is

$P_{X,Y}$

x \ y	2	3	4	5	6
1	0.1	0.1	0	0	0
3	0.1	0	0	0.1	0
4	0	0.1	0.2	0	0
6	0	0	0	0	0.3

- Find $P[X + Y < 7]$

Step 2: Add the corresponding probabilities from the joint pmf (matrix)

$P[X + Y < 7] = 0.1 + 0.1 + 0.1$
$= 0.3$

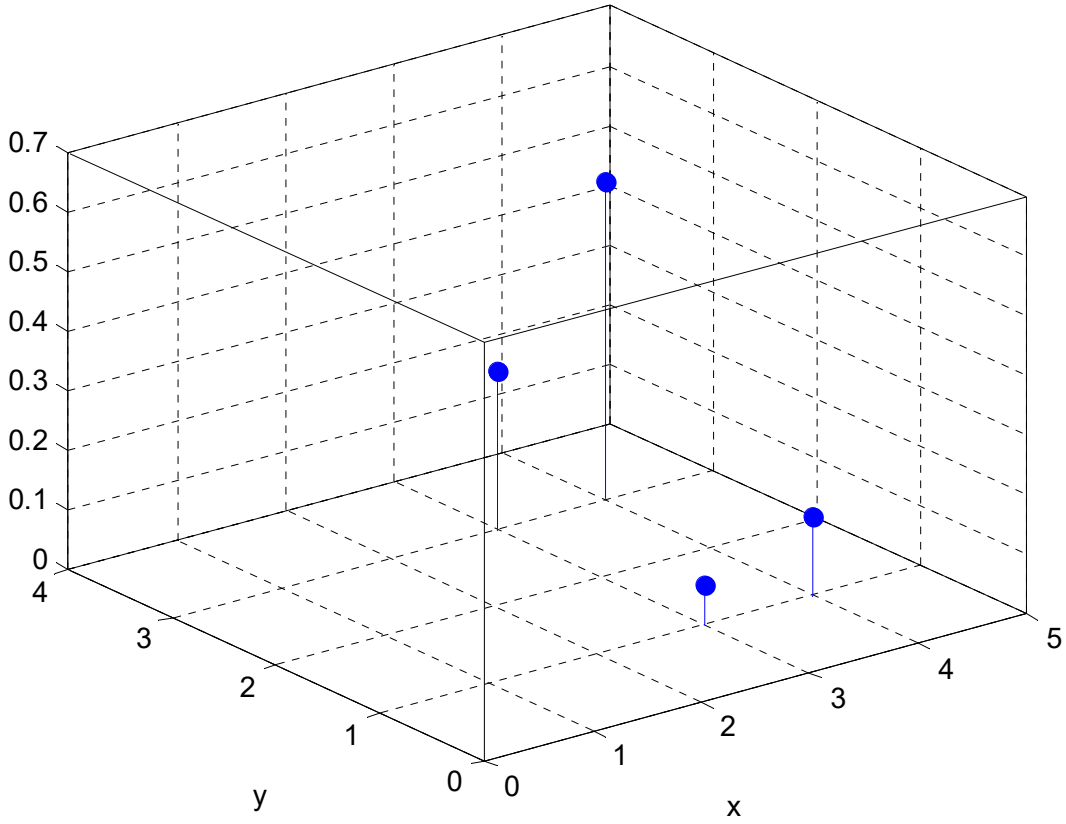
$x + y$

x \ y	2	3	4	5	6
1	3	4	5	6	7
3	5	6	7	8	9
4	6	7	8	9	10
6	8	9	10	11	12

Example: small joint pmf matrix Ex. 11.29

$$P_{X,Y} = \begin{array}{c|cc} x \backslash y & 1 & 3 \\ \hline 3 & 1/15 & 4/15 \\ 4 & 2/15 & 8/15 \end{array}$$

```
close all; clear all;
x = [3 4];
y = [1 3];
PXY = [1/15 4/15; 2/15 8/15];

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

stem3(X,Y,PXY,'filled')
xlim([0,5])
ylim([0,4])
xlabel('x')
ylabel('y')
```



Joint pmf matrix for independent RVs

Command Window

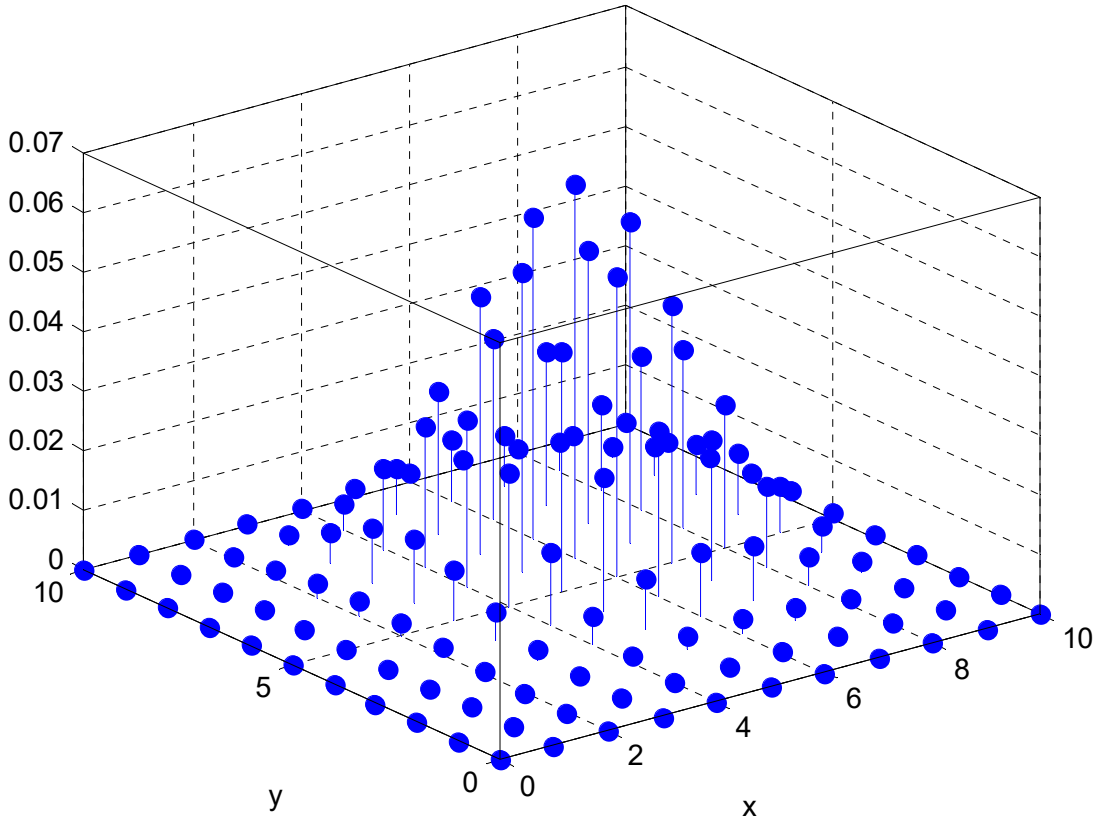
```
>> pX = [1/3 2/3]
pX =
    0.3333    0.6667
>> pY = [1/5 4/5]
pY =
    0.2000    0.8000
>> sym(pX'*pY)
ans =
[ 1/15, 4/15]
[ 2/15, 8/15]
>>
```

Joint pmf for two i.i.d. RVs

```
close all; clear all;
n = 10; p = 3/5;
x = 0:n;
y = 0:n;

pX = binopdf(x,n,p);
pY = binopdf(y,n,p);

PXY = pX.'*pY;

[X Y] = meshgrid(x,y);
X = X.'; Y = Y.';

%stem3(X,Y,PXY, 'filled')
mesh(X,Y,PXY)
%surf(X,Y,PXY)

xlabel('x')
ylabel('y')
```

Note how the pmfs are multiplied because of the independence.

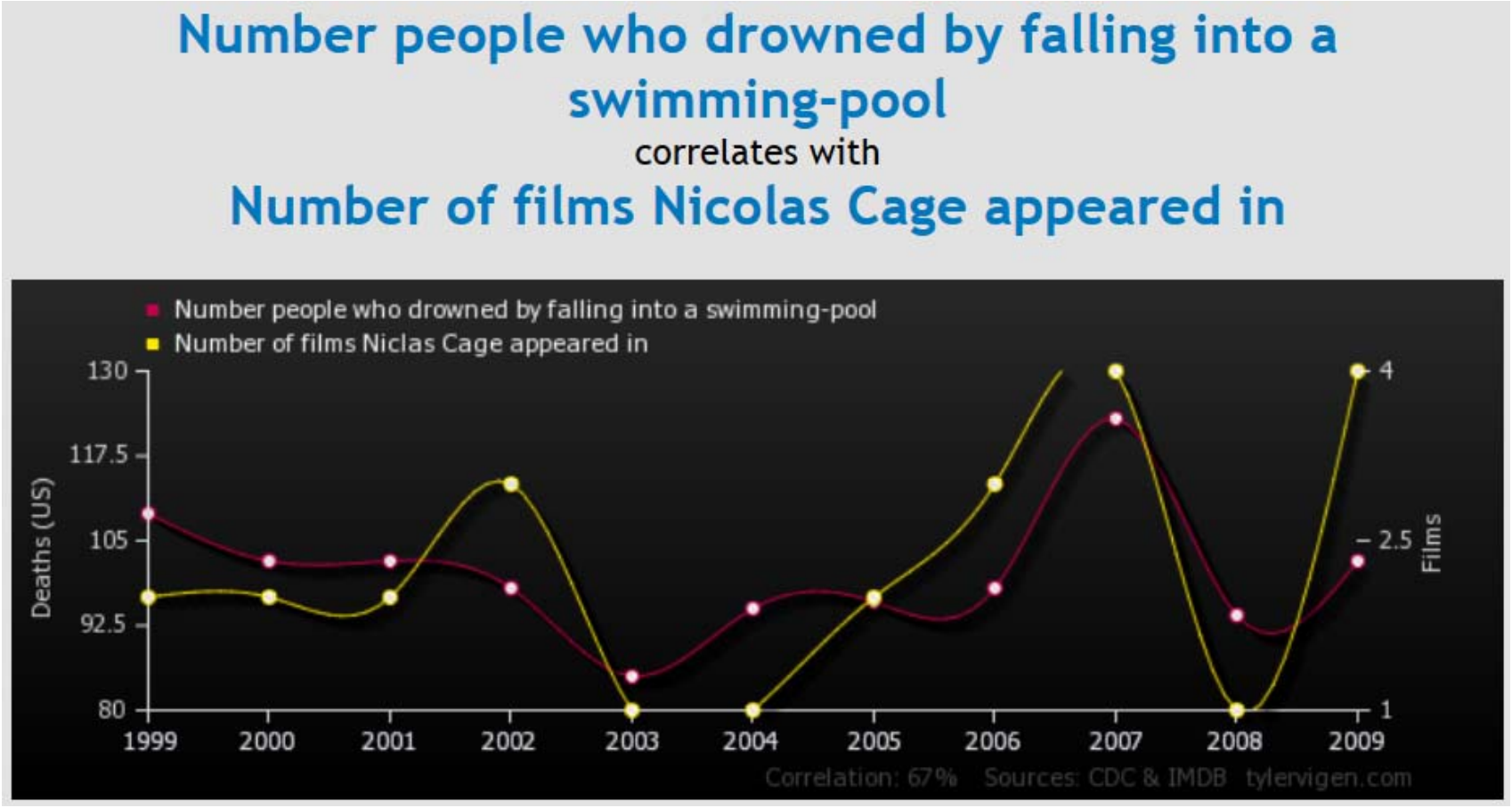
$$X, Y \overset{\text{i.i.d.}}{\sim} \mathcal{B}\left(10, \frac{3}{5}\right)$$



Correlation

- Correlation measures a specific kind of dependency.
  - Dependence = statistical relationship between two random variables (or two sets of data).
  - Correlation measures "linear" relationship between two random variables.
- Correlation and causality.
  - "Correlation does not imply causation"
  - Correlation cannot be used to infer a causal relationship between the variables.

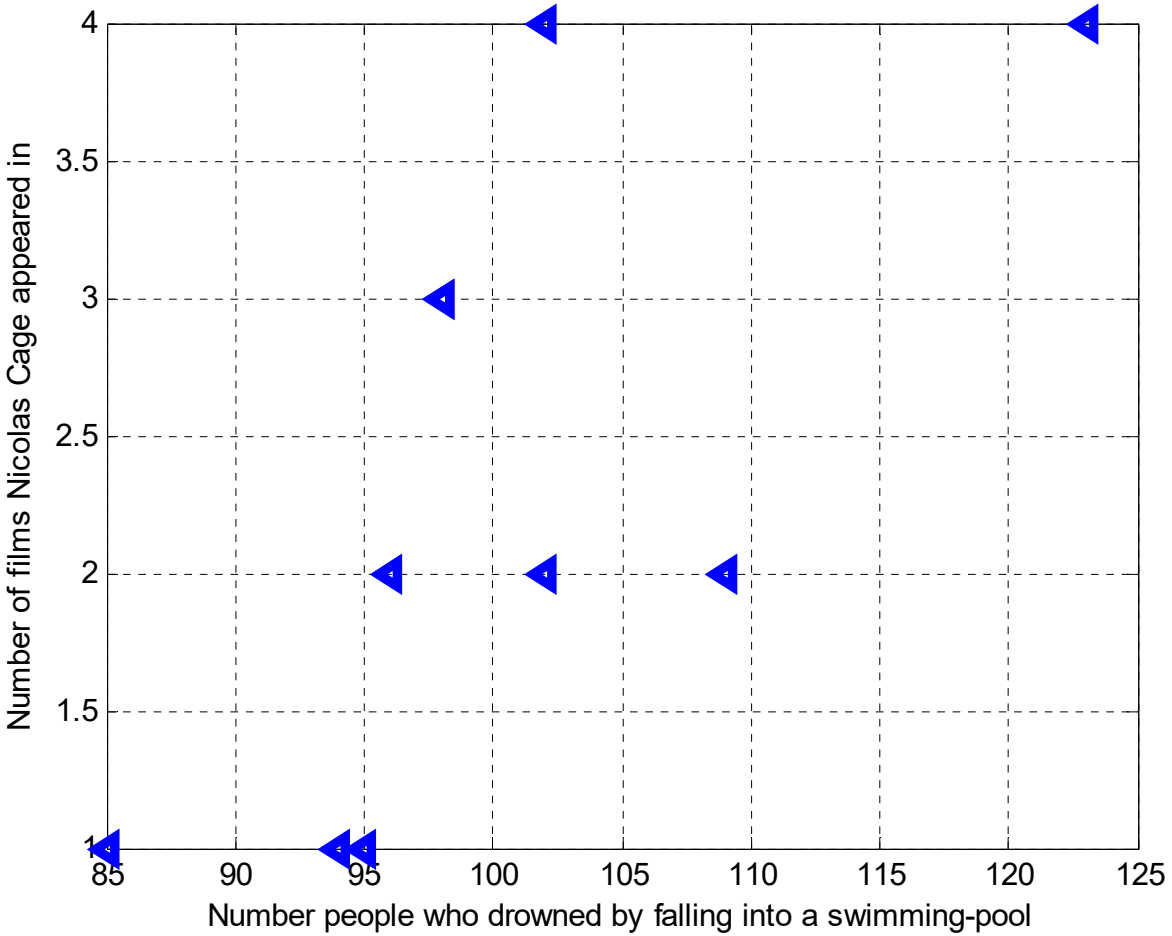
Two “Unrelated” Events



Correlation: 0.666004

http://www.tylervigen.com/

Two “Unrelated” Events



Correlation: 0.666004

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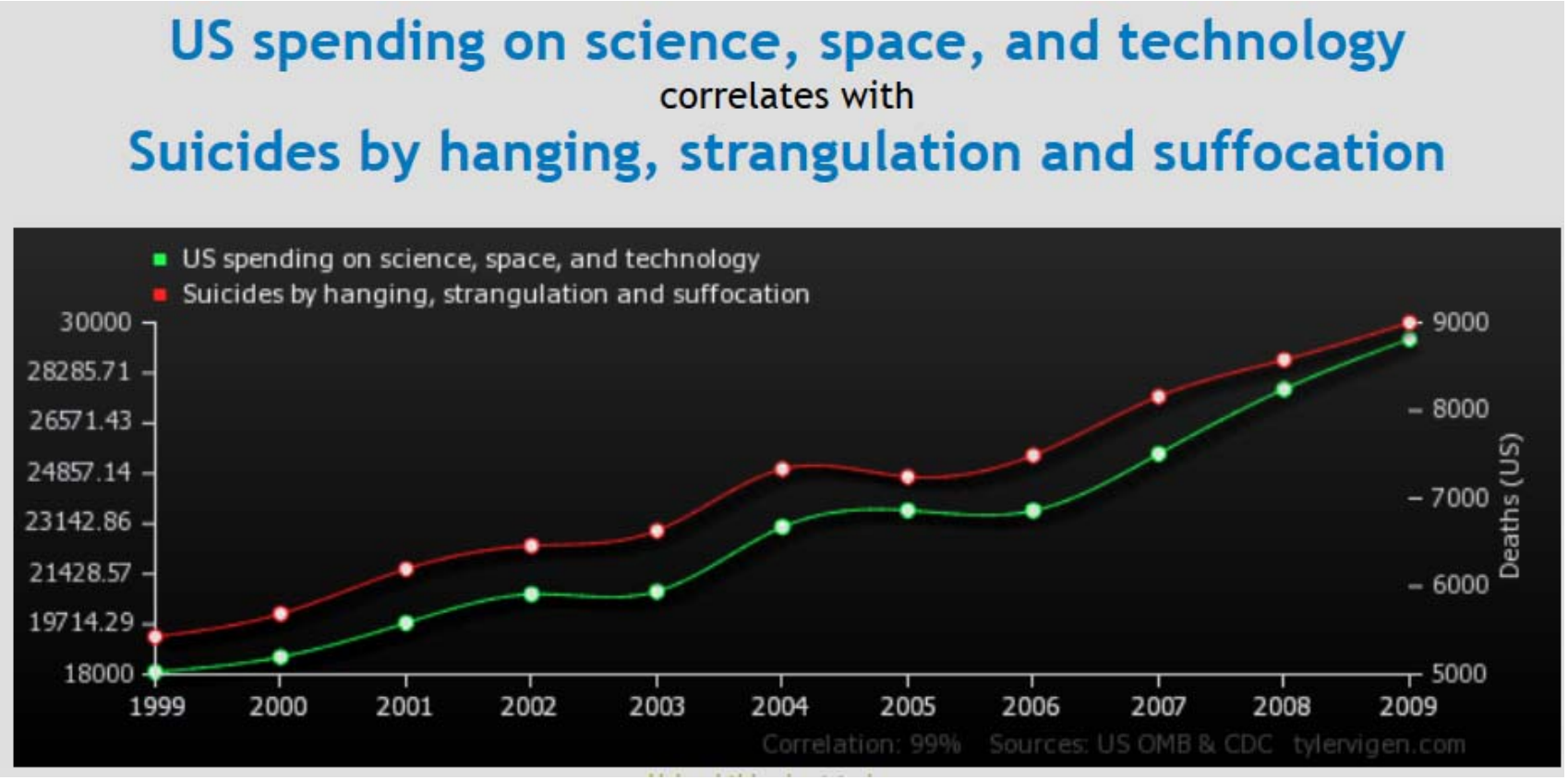
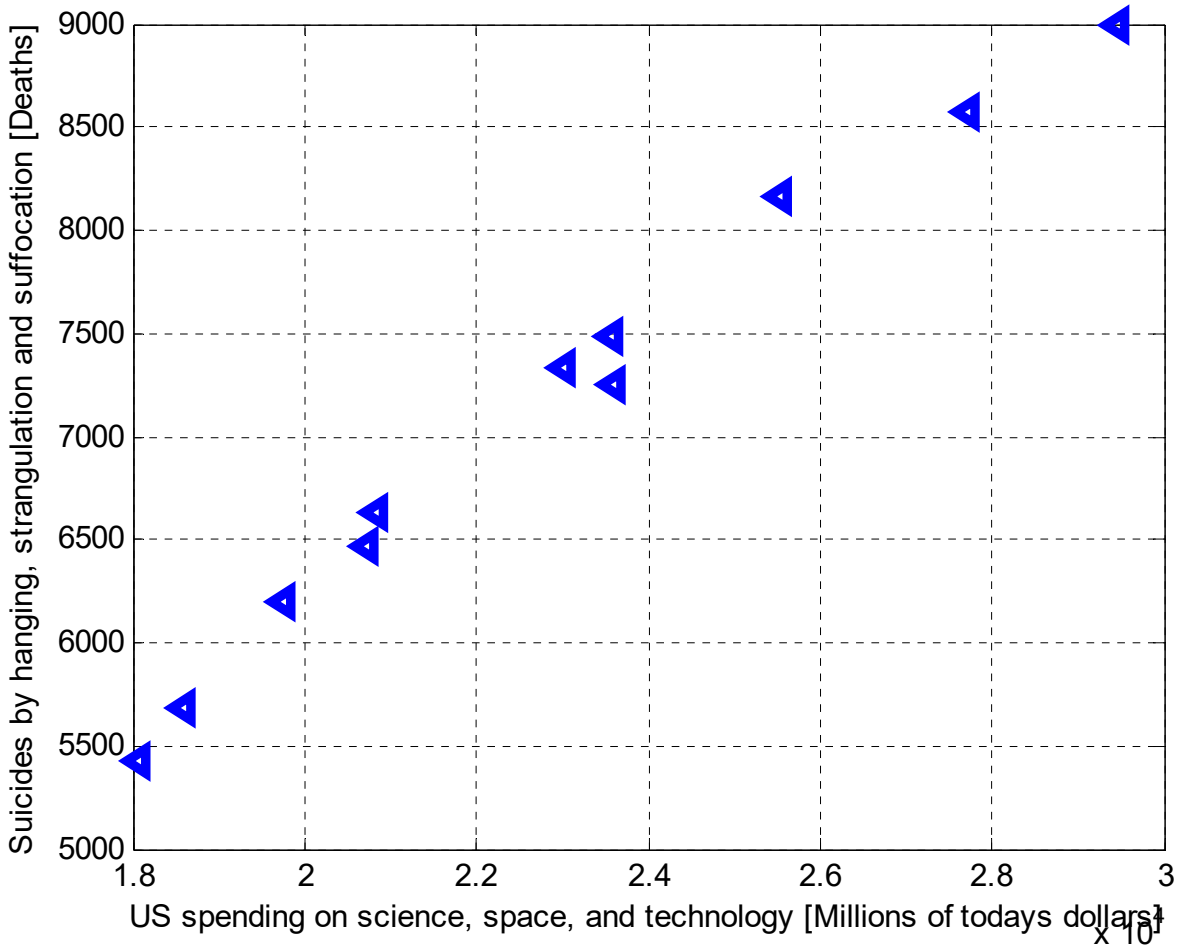
Spurious Correlation



Correlation: 0.992082

http://www.tylervigen.com/

Spurious Correlation



Correlation: 0.992082

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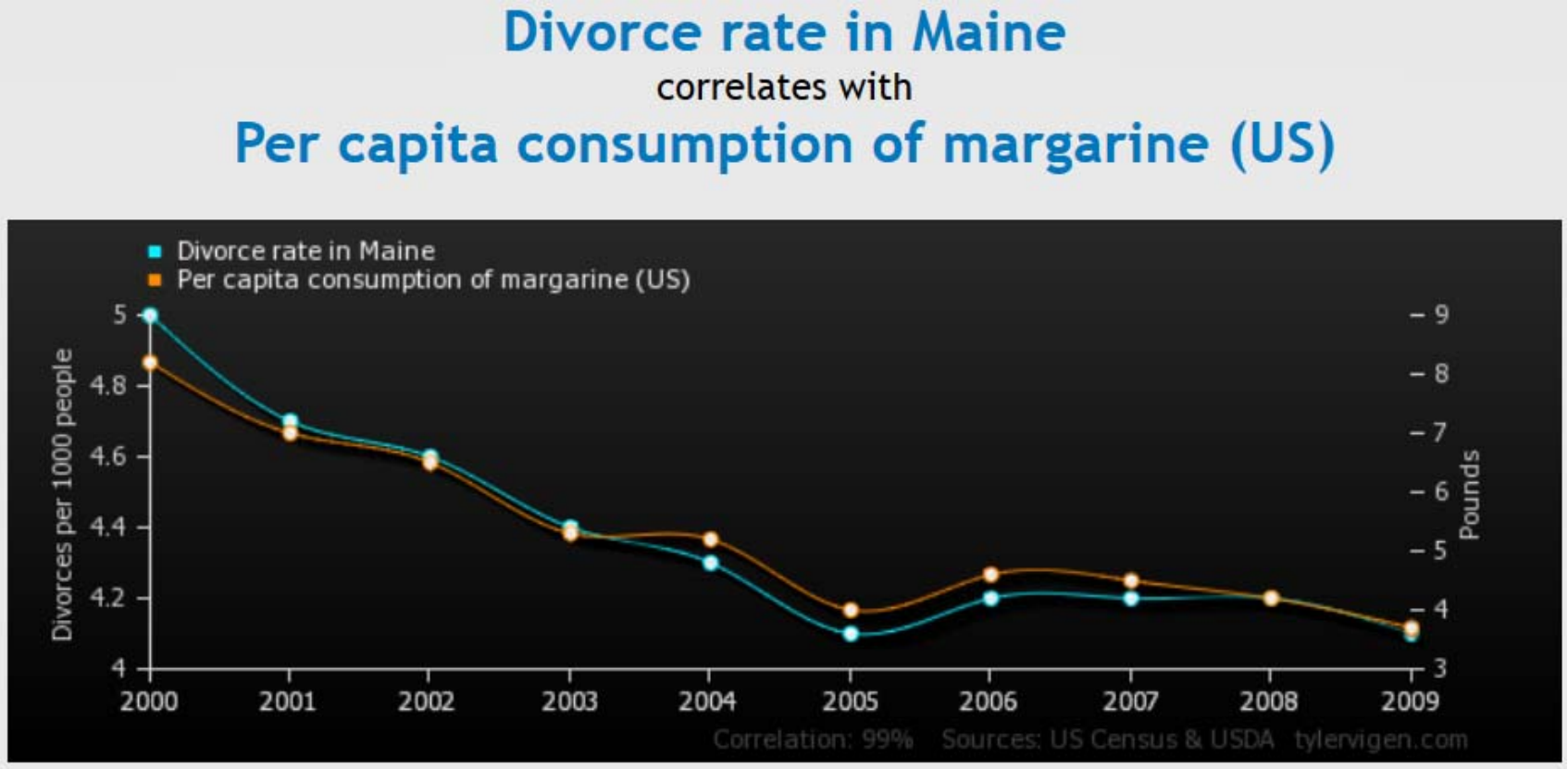
Spurious Correlation



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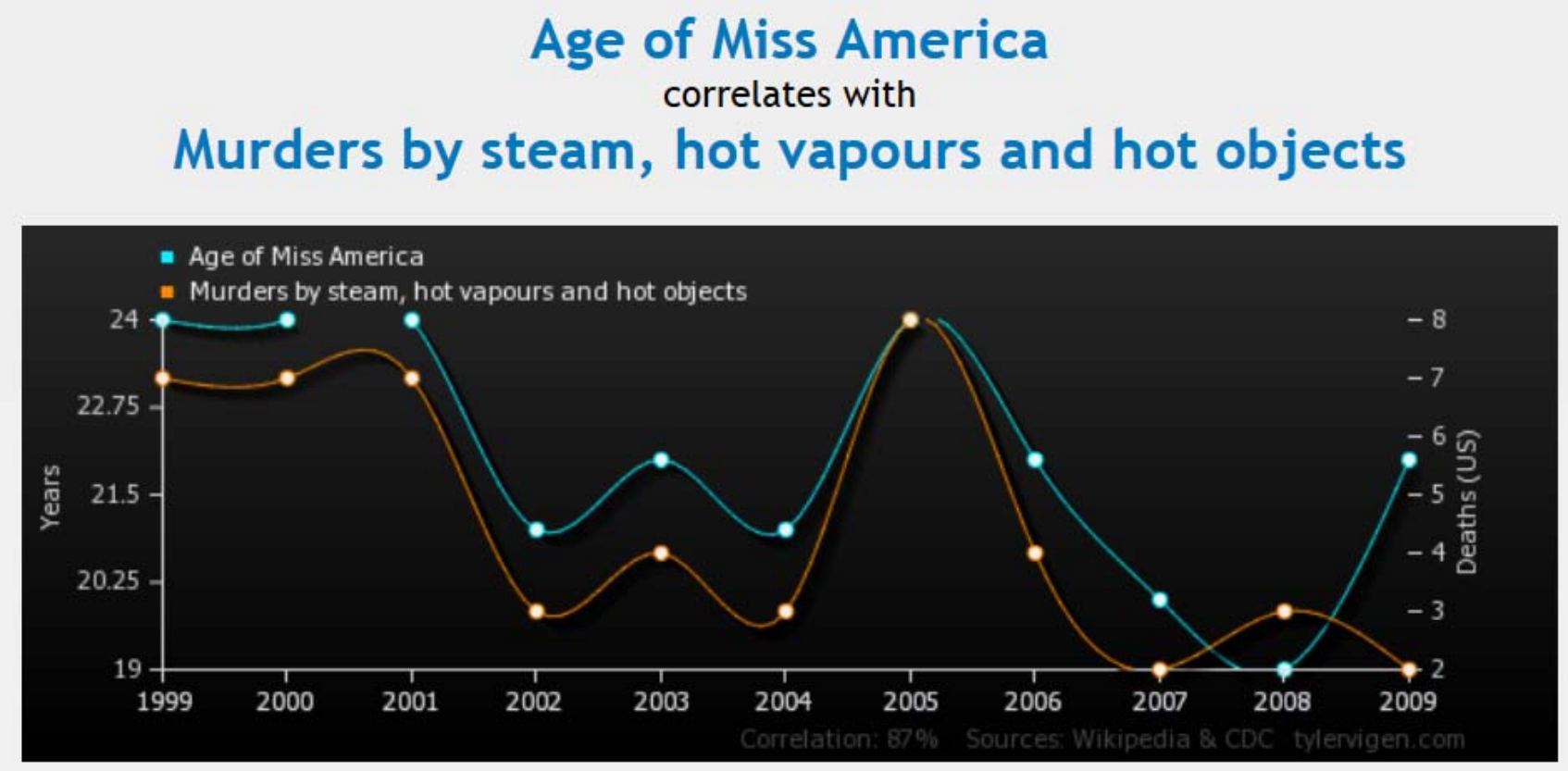
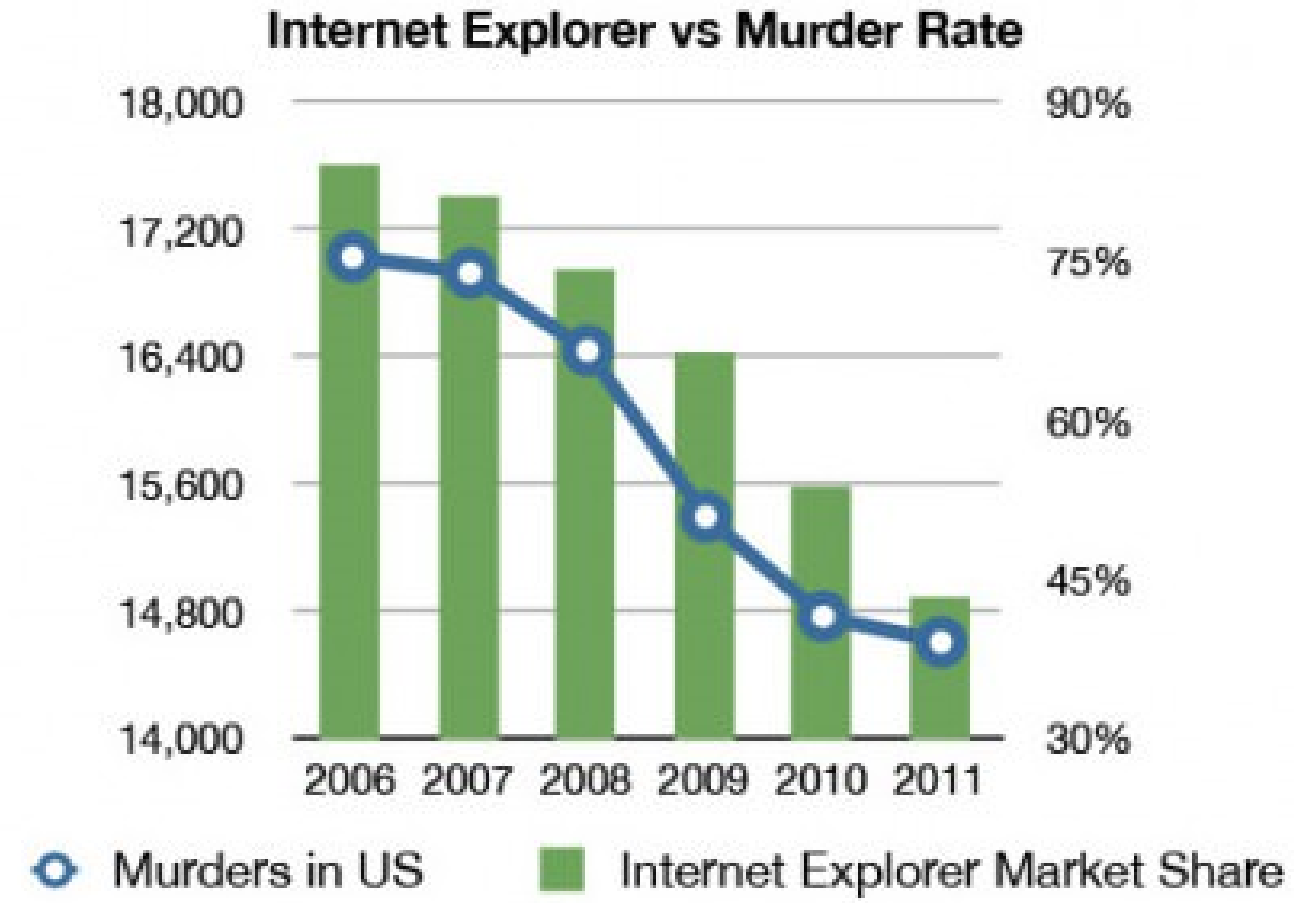
Spurious Correlation



http://www.tylervigen.com/

Spurious Correlation



(gross number of murders)

[http://www.geek.com/microsoft/does-internet-explorers-falling-market-share-mirror-the-drop-in-us-homicides-1537095/]

Spurious Correlation